An AC Bridge for Semiconductor Resistivity Measurements Using a Four-Point Probe

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A new direct-reading ac bridge circuit has been developed to measure semiconductor bulk and sheet resistivity, using a four-point (or other appropriate) probe. The range of resistivity which can be measured is from 0.001 to 10,000 ohm-cm. Resistivity is read directly from resistance decades and a ratio multiplier, eliminating voltmeter and ammeter errors — the final reading being the result of a bridge-balancing operation for each measurement. Stability and sensitivity provide better than 0.5 per cent electrical accuracy, with mechanical point spacing being the controlling limitation on the over-all accuracy of the measurement.

The use of ac eliminates the influence of rectification, thermal, or contact potentials on the measurements, and also provides sensitivity more readily than with dc. The four-point probe and test specimen are the only nongrounded elements.

An Appendix compiles four-point probe conversion factors for thin circular and rectangular slices of material. New tables are presented for slices having a continuous diffused skin all over, and thus also conducting across the back.

I. INTRODUCTION

A basic measurement made on a semiconductor material is its resistivity. This is a measure of the impurity content, and determines the suitability of the material for a particular application and the necessary process parameters for subsequent operations. This measurement also determines whether a process step has been performed satisfactorily. Present methods for making this measurement usually are variations of the basic voltmeter-ammeter circuit, using direct-current power supplies and instruments. Such direct-current methods have many causes for error, several of which, while known to exist, are difficult to evaluate.

An ac measuring circuit has been developed for measurement of resistivity, retaining the four-point probe, but eliminating or minimizing to a negligible amount the errors inherent in the former dc systems. Every component of the new system is ac operated and grounded, except the test specimen and the four-point probe. By using an ac bridge, neither current nor voltage is measured. Rather, only their ratio is read from an accurate resistance standard, which really is what is required. The circuit can also be used for dc, but then many of its advantages are lost.

The new electrical ac system is accurate to 0.5 per cent when a test current that develops at least 1 millivolt between the two voltage probes is used. It includes built-in calibration for a system check at any time. It does not correct errors in point spacing of the four-point probe, but can provide a presetting for any given spacing, so that the resistance ratio dials read directly the bulk resistivity in terms of a semi-infinite body. Also, because it is a bridge system, the balance can be servo-controlled and plotted continuously on a recorder, or read out and recorded digitally.

II. PRINCIPLES OF A FOUR-POINT PROBE MEASURING CIRCUIT

The problem can be introduced by a description of a common dc system. For a resistivity evaluation of semiconductor material, electrical connections are made by pressing four needle points against the surface of the specimen. A convenient geometry is to space equally the four points along a straight line. Measuring current is then passed through the two outer points, called the current probes, and the voltage developed thereby between the two inner points, called the voltage probes, is measured. A common dc circuit is shown in Fig. 1.

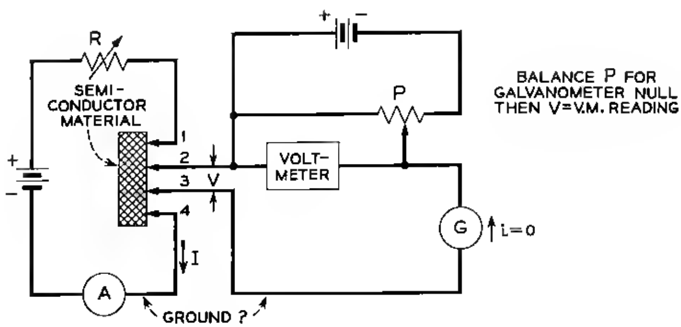


Fig. 1 — Basic four-point probe measuring circuit.

Each point introduces an unknown constriction resistance into the circuit, the minimum value of which is determined by the point pressure and the bulk resistivity of the specimen, as will be shown. In Fig. 2 is shown the equivalent resistance network of the semiconductor body and the four needle-point connections.

The unknown resistances of each of the points are shown as w, x, y, and z, respectively. The magnitudes of w and z can be compensated for by decreasing the current controlling resistor R of Fig. 1, or by using a constant-current generator instead. Thus by any of several means the current through the points 1 and 4 can be set to a specified value and measured by means of the current meter A, regardless of the magnitudes of w and z.

The voltage determination makes use of a balancing arrangement to eliminate current through points 2 and 3 with their unknown contact resistances x and y. By adjusting the potentiometer P, until the brush location is found for the condition of no current through galvanometer G, the voltage read by the voltmeter is that which opposes and exactly equals the unknown voltage V. Of course, if a high enough resistance dc voltmeter is used, the balance method can be avoided. "High" means of the order of 1000 times higher than whatever the unknown resistances x and y might be. Such voltmeters are power-line operated and require a ground on one terminal to bypass parasitic power line leakage currents away from the device being measured. One such ground can be connected to the circuit, such as on the galvanometer side. If this is done, then

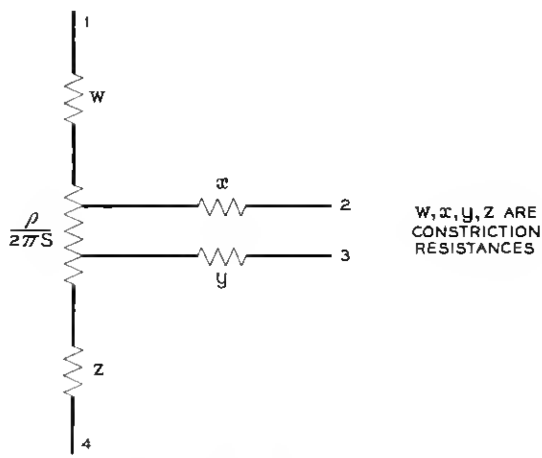


Fig. 2 — Equivalent circuit of semiconductor and four needle points.

the current supply cannot be grounded with this circuit, since points 3 and 4 would then be shorted and the current distribution in the semiconductor would no longer be known. Hence, floating batteries must be used with a power-line operated voltmeter.

Mathematical statements of the relationships involved in a resistivity determination using a four-point probe have been given by Valdes[1] and Smits.[2] For the case of a semi-infinite body and equal point spacings, the expression relating bulk resistivity, ρ, current, I, voltage, V, and point spacing, S, is

$$\rho = \left(\frac{V}{I}\right)(2\pi S).$$

Other expressions are derived for other than uniform spacing and proximity effects of nearby boundaries, all of which only alter the $(2\pi S)$ factor. In every case, the ratio of V/I appears explicitly. Thus, a determination of the voltage difference between the two inner points caused by a current flow through the two outer points is an indirect approach for a resistivity determination. Neither V nor I is really wanted, but rather their ratio. A direct measurement of this ratio is one of the features of the new circuit.

III. BLOCK DIAGRAM AND CIRCUIT FEATURES

The new bridge circuit and the use of ac rather than dc are shown in Fig. 3. The basic circuit is straightforward and inherently accurate. An oscillator in series with a current-limiting resistor sends alternating current through the two current points, the test specimen, and a grounded decade resistor. The voltages to ground of the two voltage points and part of the decade resistor are connected to three high-input impedance, nonphase-reversing amplifiers. The largest voltage, V_1, is next reversed in phase and added to the other two. The decade potentiometer is then adjusted until the sum is zero. A preamplifier, band pass filter, and an ac null detector provide the indication for this condition. The derivation of the equality of (V/I) in the test specimen to the decade resistor and potentiometer setting is given in the figure.

A summary of the advantages is as follows:

1. All components except the four-point probe and specimen are grounded.
2. The circuit is entirely ac operated.
3. Voltmeter terminals are two "open grids."
4. There are no meters, only a frequency selective null detector.

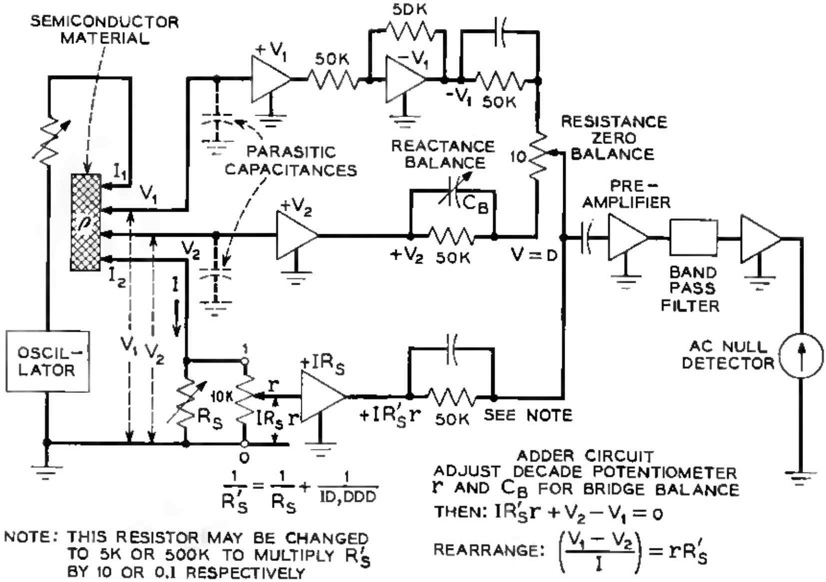


Fig. 3 — Diagram of measuring circuit.

5. Single direct reading of (V/I) from precision decade resistor and potentiometer is possible.

6. A constant-current source is unnecessary.

7. The current-resistor network can include a factor for actual probe-point spacing and read resistivity directly.

8. Servo balance of bridge can be applied for automatic measurement.

9. The circuit is independent of frequency.

The ac method also affords the following additional features:

10. Rectifying point contacts, barrier potentials and thermal potentials do not affect the fundamental ac voltage-current ratios, either in the test specimen or in the measuring circuits.

11. Superimposed DC can be used for incremental resistivity measurements, or to forward-bias a rectifying-current probe point.

IV. CONVERSION TO AC AND ELIMINATION OF METERS

For an ac system the current supply becomes an oscillator, which will be grounded. We now must measure the ac voltage between terminals 2 and 3 of Fig. 1, both of which are off ground. This suggests the use of a "differential" voltmeter. The voltages V_1 and V_2 of Fig. 3 are

ac voltages to ground, and their difference is the desired quantity. However, the voltage V_2 to ground is many times greater than the difference between V_1 and V_2, because at least that same difference exists between points 3 and 4 plus the added voltage drop in the still-present point-contact resistance z of Fig. 2 [which voltage is from 100 to 1000 times greater than $(V_2 - V_1)$], plus the drop in the decade resistor R_S. The use of part of the voltage drop across R_S, which will be found is made equal to $(V_1 - V_2)$ by adjustment, is the feature which eliminates meter readings. The three voltages are summed in an adder circuit, and when they are equal to zero as shown by the preamplifier, bandpass filter and null-detector, they yield directly the desired result:

$$\frac{V_1 - V_2}{I} = rR_S'.$$

Another important advantage is that a constant-current source no longer is needed. As the current changes, so do the voltages, and the bridge remains balanced.

The success of the circuit of Fig. 3 depends upon the high input impedance and relative linearity of the voltage amplifiers designated as $+V_1$, $-V_1$, and $+V_2$. Conventional differential voltmeters usually do not have adequate common-mode voltage suppression. Instead, precision amplifiers and a voltage adder circuit have been adopted, using the techniques of analog computers. The amplifier requirements and designs which satisfy them will be presented.

In return for ac power operation and grounding of all components except the four-point probe and test specimen, only the two indicated parasitic capacitances, in conjunction with the point-contact constriction resistances, x, y, and z of Fig. 2, contribute third-order errors.

The input impedance of each of the probes including the wire connections and "open grid" is essentially a capacitance of the order of 16 mmf. Each capacitance between each probe point and ground introduces a second-order quadrature voltage. Their difference is balanced out during each measurement by use of the reactance balance. There remains only a third-order error in the resistivity reading itself, which places a limit on the test frequency-resistivity product, as will be shown. Because of this, material below 100 ohm-cm resistivity is measured with a four-point probe and 390-cycle test current. Up to 500 ohm-cm resistivity, still with a four-point probe, the test frequency must be lowered to 85 cycles, which then also requires the use of a 4-cycle bandwidth wave analyzer or equivalent, for null indication. For material from 500 to 10,000 ohm-cm resistivity the two-point probe method, with end-

plated current connections and 85 cycles must be used. This is because surface states produce, in addition to capacitance-current effects, a nonhomogenous structure, and the curvilinear current flow no longer can be defined.

V. SUMMARY OF COMPONENT REQUIREMENTS

A description has been given in general terms of a semiconductor ac bridge resistivity measuring circuit. Aside from the oscillator, test specimen, and four-point probe, it consists of components having the following requirements:

1. A decade resistor and potentiometer network.
2. High input impedance precision voltage amplifiers, with (a) required input resistance of the order of 50,000 megohms and (b) relative linearity of one part per million.
3. A voltage adder, with adjustment for a stable zero balance of one part per million.
4. A high-gain selective null detector, which will (a) reject 60 cps and its harmonics; (b) reject harmonics of the testing frequency, primarily the second; and (c) have sensitivity of 0.5 microvolt.

The following sections of this paper will consider each of these parts in detail, to arrive at the component requirements in terms of the measurement accuracy objective. The procedure is to make each part capable of 0.1 per cent accuracy. Then a 0.5 per cent over-all accuracy will be realized.

VI. DECADE RESISTOR NETWORK

A slightly more detailed current-resistor network circuit is shown in Fig. 4. A precision 10,000-ohm Kelvin-Varley decade potentiometer calibrated as a ratio from 0 to 1 is in parallel with a preset fixed resistor decade. Once the decade has been set, the circuit balancing by the potentiometer will not alter the ac through the specimen.

For the decade resistors, the smallest step is 0.01 ohm and the largest is 100 ohms. With the low-resistance steps used with low-resistivity material, the dial-indicated resistance is not very accurate, primarily because of resistance in the wiring connecting it to the circuit. This wiring is fixed and can be measured, and a calibration chart can be made for each test set. This value is further subdivided into 10,000 parts by the decade potentiometer. Equal accuracy thus obtains for any decade resistor setting.

The resistance decades are connected in parallel with the ratio decades,

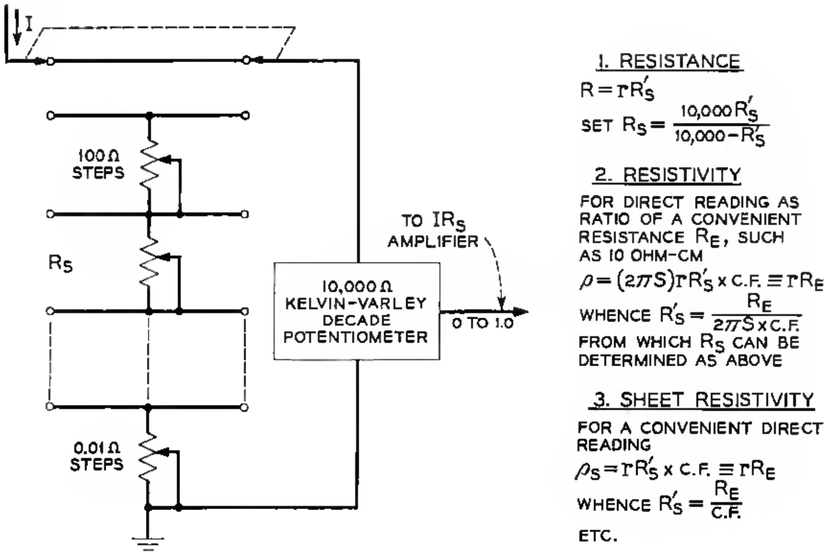


Fig. 4 — Decade resistor and potentiometer network.

A computation has to be made to determine the decade resistor setting to provide the desired parallel resistance. For instance, if 1000 ohms were wanted, the decade resistors would be set at 1111.1 ohms; if 10,000 ohms were wanted, they would be disconnected.

The resistor decade setting can be chosen to have the potentiometer read:

(a) resistance directly;

(b) body resistivity directly, including the actual point spacing; or

(c) sheet resistivity directly, including the actual point spacing.

The determination of R_s for these methods of operation is shown in Fig. 4.

VII. SEMICONDUCTOR EQUIVALENT NETWORK AND DIFFERENTIAL AMPLIFIER REQUIREMENTS

The requirements for the differential voltmeter are determined by the semiconductor material itself. To show this, an equivalent network of the system consisting of the material with the four needle points in contact, is needed.

Referring to Fig. 2, we need to know the order of magnitude of constriction resistances w, x, y, and z. For a given specimen, they all will be somewhat alike, but we cannot make the assumption that they are

even known, or equal. An important way in which the resistance z enters can be seen by the following.

There is a voltage drop at each current probe caused by the current constriction, the effect appearing as though there were resistances w and z as in Fig. 2. Similarly, there appear resistances x and y in the voltage probes, even if there is no current flowing. The resistance z is of first concern, though compensated for as regards current flow by the external circuit, because a voltage across z must be suppressed by the differential voltmeter. It will next be shown that this voltage drop to ground is of the order of 100 to 1000 times higher than the wanted voltage difference between the voltmeter probes 2 and 3.

Holm[3] has developed an expression for the constriction resistance between two materials in contact, one of them yielding plastically,* as

$$R = 0.445\rho \left(\frac{P_y}{F}\right)^{\frac{1}{2}} \text{ ohms},$$

where

ρ = resistivity in ohm-cm of the higher resistivity material,

P_y = yield or tensile strength in grams/cm^2,

F = contact force in grams.

Experimentally it has been found that silicon undergoes only elastic deformation before fracturing. In the 111 plane the fracture strength is about 20×10^6 grams/cm^2. No plastic yield strength data for osmium, the probe point material, have been determined, but they are estimated to be of the same order as those of steel, about 14×10^6 grams/cm^2. As this latter figure is lower, its plastic flow can be assumed to control the contact area. Thus, if each needle point has a force of 25 grams, then

$$\frac{R}{\rho\,\text{cm}^{-1}} \doteq 300 .$$

This ratio will change inversely as the square root of the applied contact force.

This is the minimum effect that has to be anticipated. Films may increase the ratio further, and rectification will introduce other erratic effects which will act to increase the effective resistance as well as to

* From Equation (14.08) of p. 75, of Ref. 3, combined with Equation (15.01), p. 79. Note that Holm's expression is twice the above, because his case is for like materials, whereas the resistivity of the osmium point used in the probe is small compared to that of a semiconductor.

introduce "noise" into the measurement. Thus, we see that an optimistic equivalent network will be as shown in Fig. 5. For a wanted voltage of one millivolt, there will be a voltage to ground on each differential voltmeter input of from 0.2 to 1 volt. An error in suppressing one volt, by one part in a million, can cause up to a 0.1 per cent error in the measurement. This requirement applies regardless of the resistivity of the material—whether it is 0.001 or 10,000 ohm-cm—as long as a four-point probe is employed, because about one millivolt will always be needed for the wanted signal. This will be discussed later when thermal noise limitations are considered.

The meeting of the linearity requirement is easily checked. The IrR_s input of Fig. 3 is made zero by setting r to zero. The V_1 and V_2 input leads are connected together and to 2 volts ac. They thus have identical voltages to ground and zero voltage difference. For this test condition, there must be less than about one microvolt output into the null detector. This suppression is set up by a micro-adjustment of one adder low-resistance potentiometer called the *resistance zero balance*. Then the oscillator output is decreased toward zero voltage and no output voltage should appear to upset the null balance. A switch is provided for conveniently making this bridge balance and testing for linearity. Input

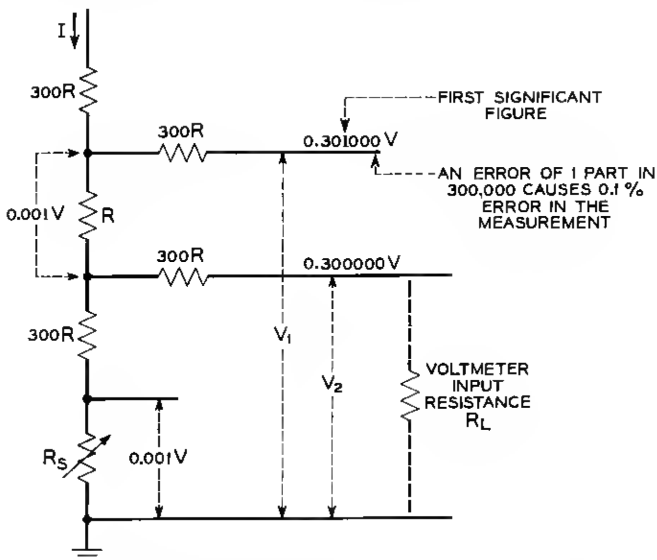


Fig. 5 — Differential amplifier linearity and input resistance effects.

tubes for the amplifiers have to be selected to meet this test. About one-third of those tested meet this requirement.

A second effect is the input resistance component of the differential voltmeter, including the vacuum tube socket, the grid and grid current, switches, wire insulation, leakage of the probe points to the supporting frame, etc. The requirement here is a function of the resistivity being measured. On Fig. 5, a spurious resistance is shown as R_L , connected between probe V_2 and ground. Suppose, for convenience, that a silicon sample of 500 ohm-cm resistivity is being measured and $2\pi S = 1$. Each probe contact will be of the order of 150,000 ohms. To first order, the 0.3 volt to ground at the inaccessible internal junction will be reduced at the accessible probe terminal V_2 by the amount

$$\Delta V_2 = \frac{0.15 \times 10^6}{0.15 \times 10^6 + R_L} \times 0.3 \text{ volt}$$

by ordinary potentiometer action. This is a direct error voltage and for 0.1 per cent accuracy must be less than 10^{-6} volt. For example, 10^{-3} volt is as large as the voltage we are attempting to measure. For 500 ohm-cm material the above equation shows that we must have

$$R_L > 45{,}000 \text{ megohms.}$$

For lower resistivity material, of course, this number becomes lower, but even for 1 ohm-cm material it is 10^8 ohms. It is obvious that only an "open grid" differential amplifier will be adequate. This, and the use of short grid leads, polyvinylchloride insulation, ceramic high-insulation switches, clean thermo-setting plastic mounting for the probe points, and point-to-point wiring, realize the requirement.

Thus, the second requirement for the differential amplifier is that it must have an input resistance greater than 50,000 megohms. This has been achieved.

However, we are using ac, and the reactance of the parasitic capacitance to ground of the differential volt-meter leads may be much less than the above. This places a limit on the resistivity frequency product which can be used. This limit will be developed in a later section. Basically, we can anticipate the results by observing that such a reactance primarily will produce 90° phase shift currents. Voltage drops due to such currents can be identified and balanced out, leaving only the wanted in-phase voltage.

VIII. NULL DETECTOR SENSITIVITY AND BANDWIDTH

The required voltage sensitivity of the bridge amplifier is a problem in signal to noise ratio. If the voltage probes have 1 millivolt difference,

then the voltage from the IrR_s amplifier is also 1 millivolt at the balance condition. An error of 0.1 per cent in setting tbe balancing voltage develops 1 microvolt change, which appears at the input of the null amplifier as about $\frac{1}{3}$ microvolt, because of the potentiometer action of the adder network. The presence of such a voltage must be recognized for an over-all accuracy of 0.5 per cent.

An ultimate limitation is thermal resistance and tube noise. The lowest signal voltage point in the circuit is the input to the null amplifier. With receiver type tubes, the circuit impedances can be kept to about 20,000 ohms. The average thermal resistance noise voltage is given by the formula:

$$E = \sqrt{4RKT\Delta F},$$

where

$$\begin{aligned} R &= \text{resistance in ohms,} \\ K &= \text{Boltzman's constant, } 1.38 \times 10^{-23}, \\ T &= \text{temperature in degrees Kelvin,} \\ \Delta F &= \text{bandwidth in cycles per second.} \end{aligned}$$

Assuming a 100 cps bandwidth selective filter, 20,000 ohms resistance, and a temperature of 310°K, the average input noise voltage to the bridge amplifier will be

$$\begin{aligned} E &= \sqrt{4 \times 2 \times 10^4 \times 1.38 \times 10^{-23} \times 310 \times 100} \\ &= 0.2 \text{ microvolt.} \end{aligned}$$

Generally, the first tube plate contributes noise of the same order referred to the input, and other sources must be allowed for. These will be random. With careful design we can anticipate an over-all average noise of about 0.5 microvolt. Adding an error signal of this same amount should give an easily perceptible signal, unless the random nature of the circuit noise and its high peak factor cause too much instability in the no-signal indication. Even this can be alleviated to a considerable extent by damping the dc meter winding with a very large capacitor. The wanted signal is a steady sine wave so, after rectification, damping will not affect it but will suppress the occasional noise peaks to their time average value.

Most of the common laboratory ac voltmeters have a full scale sensitivity of 10 millivolts or better. One tenth of this, or 1 millivolt, can readily be discerned. The null preamplifier gain, from input to filter

output, therefore must be about 2000. The filter midband frequency, which has not yet been mentioned, will be arrived at in a following section, analyzing the effect of probe parasitic capacitance; 390 cycles has been adopted for all doped device material, but 85 cycles must be used for floating zone refined silicon.

Another source of thermal resistance noise is the constriction resistance at the probe points V_1 and V_2. These become controlling with resistivity material of about 50 ohm-cm or higher. The solution for this situation is to use a 4-cycle bandwidth for the null detector or a larger test current to develop 10 to 100 millivolts between the voltage probes. With high resistivity material, the power dissipated is negligible. Both of these means can be used.

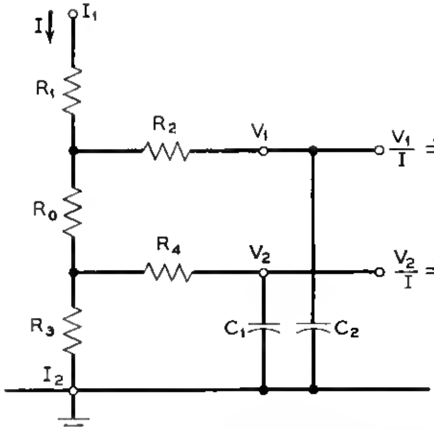
IX. BRIDGE REACTANCE ZERO BALANCE

The circuit of Fig. 3 shows the adder network to consist of three equal resistors. Two of them, the $-V_1$ and the $+V_2$ are matched to one part in a million through the use of the resistance zero balance. Second, a reactance adjustment is also necessary to mop up for parasitic capacitance effects in the wiring of the amplifiers and adder circuit, even though the resistivity of the test specimen is quite low, so the probe capacitances are unimportant. The additional considerations for high resistivity material will be covered in Section X. For an extreme adjustment range of ± 50 mmf, it can be shown that an error of less than 0.07 per cent is introduced when a test frequency of 390 cycles is used.

X. ERROR DUE TO PARASITIC PROBE CAPACITANCE

As mentioned earlier, with high-resistivity material the quadrature currents through the voltage probe parasitic capacitances to ground introduce voltages to ground which are larger than the wanted voltage difference, and sometimes too large for compensation by use of the bridge reactance balance. Fortunately, just as the adder network subtracts the two in-phase ground voltages to develop the wanted voltage difference, so does the adder network subtract the two quadrature voltages. Variable trimmer capacitors can be added across each voltage probe to ground, one of which being used during a measurement to increase the smaller of the two quadrature voltages. However, even though the second-order reactance component is balanced, there is a third-order error term remaining which places a limit on the resistivity-frequency product for a specified error.

An appropriate equivalent circuit for analysis is shown in Fig. 6.



$$\frac{V_1}{I} \doteq \frac{R_0+R_3}{1+j\,\omega C_1 R_2\left[1+\frac{R_1}{R_2}\,\frac{R_0+R_3}{R_0+R_1+R_3}\right]}$$

$$\frac{V_2}{I} \doteq \frac{R_3}{1+j\,\omega C_2 R_4\left[1+\frac{R_3}{R_4}\,\frac{R_0+R_1}{R_0+R_1+R_3}\right]}$$

EXPAND EACH AS A SERIES, SUBTRACT, EQUATE SECOND ORDER QUADRATURE TERM TO ZERO, SOLVE FOR C_2, AND SUBSTITUTE. THEN:

$$\frac{V_1-V_2}{I} = R_0\left[1+\underbrace{\left(\omega\, C_1 R_2\right)^2\,\frac{R_0+R_3}{R_3}\left[1+\frac{R_1}{R_2}\,\frac{R_0+R_3}{R_0+R_1+R_3}\right]^2}_{\text{ERROR TERM}}+\cdots\cdots\right.$$

Fig. 6 — Error expression for probe parasitic ground capacitance.

Assuming no interaction between the second-order quadrature currents, expressions for the error term can be derived for two cases, first for a four-point probe (which up to here has been the only one mentioned) and second, a two-point probe where the current connections are made through plated terminals on a bar of material, and only the voltage probes are placed on the sample. The latter is present practice with floating-zone-refined silicon, and there are several reasons to continue this practice. In effect, this arrangement makes R_3 (and R_1) the same order as R_0 and lowers the voltages V_1 and V_2 to ground by a factor of about 0.01. This eases considerably the performance required of the amplifiers. A second effect is that rectification at the current probes is diminished. Thirdly, with high-resistivity material surface-state effects become increasingly important and cause the structure to be nonhomogeneous, invalidating an assumption made in deriving the conversion factors for resistivity expressions. At least, with current plates on the bar end the potential gradient is unaffected by surface states. There remains an error in computing current density. Near the surface it will differ from that in the interior, increasing in effect as the resistivity increases.

To attack the test frequency problem, the error expression derived

on Fig. 6 provides the answer. The easiest approach is to consider the two-point probe first. In this case, the values of R_0 and R_3 are of the same order and two orders of magnitude smaller than R_2 or R_4. For 10,000 ohm-cm material and 10 grams point force, a test frequency of 85 cps will cause an error, due only to frequency, of one per cent. Thus it will be necessary with such material to increase the point force to about 100 grams. For 3000 ohm-cm with 10 grams, the error reduces to 0.1 per cent. Thus it is clear that a frequency of this order is necessary, as well as being desirable for other reasons. The second-order error term on Fig. 6 does not indicate how accurately the trimmer condensers must be set for the quadrature voltage balancing—the lowest frequency possible is desirable for ease with this adjustment. Further, it is necessary to exclude 60 and 120 cps pickup and still detect 0.5 microvolt of wanted signal. We have shown earlier that a very narrow bandwidth is also necessary to keep the thermal resistance noise within bounds.

For device work, the bulk resistivity seldom will exceed 100 ohm-cm, permitting a much higher testing frequency and a four-point probe. For instance, assuming R_1, R_2, R_3, and R_4 are equal and 1000 times larger than R_0, substitution in the equation of Fig. 6 shows that, with 390 cycles, a resistivity of 500 ohm-cm is tolerable. However, difficulty in setting the quadrature balance and instability in the constriction resistances R_2 and R_4 have indicated a maximum of about 100 ohm-cm for this arrangement.

For zone-refined silicon, the four-point probe and 85 cycles can be used up to the order of 500 ohm-cm material, if surface-state effects are known to be unimportant. In case of doubt, the two-point probe method can always be used.

To summarize the discussion of measuring frequency and resistivity:

Four-point probe

up to 100 ohm-cm	390 cycles
up to 500 ohm-cm	85 cycles

Two-point probe

up to 500 ohm-cm	390 cycles
up to 10,000 ohm-cm	85 cycles

XI. RECTIFICATION

The connection to the semiconductor material, through the four pressure points, results in rectifying contacts. The test current flows

through the I_1 and I_2 contacts, but always in opposite sense. Thus, first one and then the other is backward biased, on alternate portions of the ac testing current. The total resistance of the series circuit, however, tends to remain constant as first one point and then the other is high resistance, and this effect is further aided by use of a relatively high series resistance in the oscillator output. The purpose of this resistor is to maintain substantially a sine wave testing current. Minor distortion is unimportant, since the wave filter in the null detector selects the fundamental frequency and excludes the distortion products.

The voltage to ground at each of the two voltage probes, however, does show the full rectification effects of only the I_2 probe point. The two rectified voltages are not quite equal because of the semiconductor body voltage drop, and this difference is the wanted sine wave for comparison to the IR_s voltage.

After transmission of the two probe voltages through the amplifiers and to the adder circuit, the subtraction which takes place in the latter largely balances the rectification components. Perfect balance to the distortion is not attempted, nor is it necessary. The wave filter selects the fundamental and rejects the remainder. Thus, in addition to noise suppression, the wave filter performs a second and equally important function. This is the reason an oscilloscope is sometimes not adequate as a null detector. As the balance is approached, the residual rectification products and noise become relatively large. Small changes in the fundamental cannot be identified.

XII. SUPERIMPOSED DIRECT CURRENT

As mentioned earlier, there is a dc path through the test specimen and oscillator circuit. Thus superimposed dc can be used. One such use is to eliminate a rectified waveform from appearing at the voltage probes. By connecting a battery or dc power supply in series with the oscillator having a magnitude slightly greater than the peak-to-peak ac voltage, the total test current through the I_1 and I_2 probe points will not reverse during a complete cycle of ac. The polarity of the dc, of course, must be in the direction to cause the I_2 probe point to be forward biased.

More direct current than the minimum can be used and incremental resistivity data taken to as high a current as desired, within the heating limitations of the semiconductor material. Most of the heat is generated directly under the back-biased probe point.

XIII. AMPLIFIERS

Each of the four precision amplifiers shown in block diagram form in Fig. 3 is a three-stage feedback amplifier. The three "diverted current"

nonreversing amplifiers are of one design and use series-input-shunt-output negative feedback. A schematic is shown in Fig. 7. The fourth, a reversing amplifier, uses shunt-input-shunt-output feedback. The internal amplification for each is 100,000, or 100 db, but externally they behave like a unity gain circuit. The internal full-gain bandwidth is from 40 to 400 cps. The internal frequency cutoff for the high frequencies starts at 700 cps and provides a constant 30° phase margin, including the effect of 120 mmf capacitance between the conductor and inner shield of the four-foot cable, and 2000 mmf between the inner and outer shields. Gain crossover occurs at about one-half megacycle. For the low-frequency end, somewhat reduced amplification extends to dc. This avoids the need for low-frequency-cutoff coupling networks. Such a circuit also carries the full probe voltage signals through to the adder network, including the dc component when rectification is present. Only after the subtraction is the residual dc error suppressed.

The input tube for all amplifiers is the 6AK5, having a gold-plated grid. The dc grid current measures less than one millimicroampere. A grid circuit is inherently a constant current circuit—hence its ac resistance is much higher than a simple dc voltage-current computation would indicate. A low reverse-current silicon diffused-junction diode is back-biased by a few volts, and connected to the grid of the second stage. This provides symmetrical overload conduction at that point, the grid itself for positive polarity and the diode for negative polarity. This arrangement prevents a Nyquist stable oscillation from occurring when the input grid is opened, causing the amplifier to go into overload, and then reconnected.

XIV. OSCILLATOR AND CURRENT SUPPLY

The oscillator should be of the bridge-stabilized type for good waveform, frequency, and amplitude stability.

The current output requirements cover a wide range. The load requires constant voltage, but the range of resistivity is expected to vary from 10,000 ohms to 0.001 ohm and the current from 0.1 microampere to 300 milliamperes. The load power in the semiconductor will be less than 1 milliwatt.

An open-circuit voltage of the order of 2 volts and an internal impedance (without feedback) designed for about a 5 ohms load is required. A series resistor of that amount will limit the current for the lowest resistivity condition and maintain good waveform. For any higher resistivity, simply increasing the series resistance reduces the test current as needed, keeping the open circuit voltage constant.

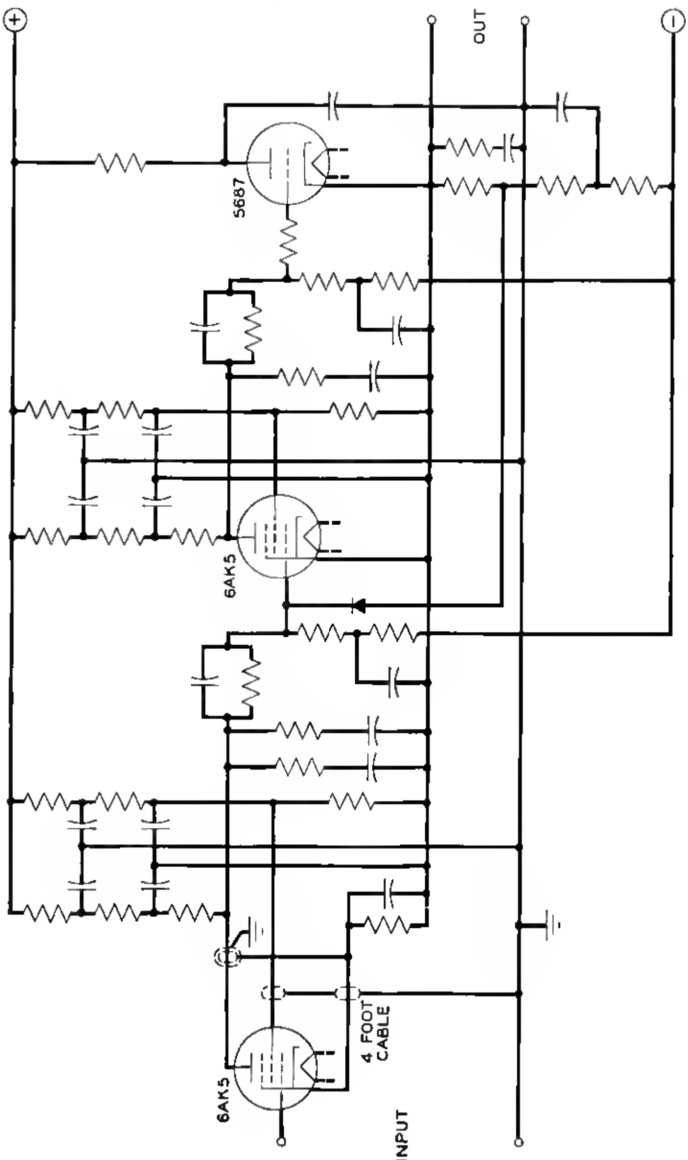


Fig. 7 — Schematic of series input one-to-one amplifier.

XV. ACCURACY AND PRECISION OF AC VERSUS DC

A statistical analysis of repeated measurements on the same specimen can establish the precision of a measurement but not the accuracy. The verification of the electrical circuit design accuracy has been established by the use of precision resistor networks such as the simulation shown in Fig. 2, including relative variations of resistances x and y by factors of 3 or 4 to 1. One of these is permanently wired to a switch, and can be used at any time. Other easily made ac tests using semiconductor material, such as varying the test current and frequency or measuring with the surface lapped or optically polished, showed slight effect using ac, validating the principles of the design, and indicate that an electrical accuracy of 0.5 per cent has been realized.

15.1 *Two-Point Probe*

The first two sets of measurements presented here have all been made using a two-point probe with the current connections made to plated end-surfaces of rectilinear bar samples. This type of measurement has no point-spacing error, and exhibits only electrical measurement errors. The two voltage points are cast in a rigid thermosetting plastic bar and pressed against the test specimen by an inverted pivot midway between the two points, assuring equal and independent point forces.

Measurements made by C. L. Paulnack and W. J. Thierfelder are shown in Figs. 8 and 9 for a "21 ohm-cm" p-type silicon bar. Note the expanded scale used for the ac chart, compared to the dc chart.

By a switching arrangement designed to avoid any mechanical motion, a forward and reverse measurement was made with the dc test set; then an ac measurement was made. The points were then lifted and reset on the surface before the next set of ac and dc measurements was made, repeating the procedure until a series of five measurements was completed each day.

After seven days of measurements, the surface of the silicon became pitted, and the points were moved 0.015 inch to a new position. The precision of the ac measurement clearly distinguished the nonuniformity of the specimen, whereas the dc test set was unable to. The dc resistivity indication was 5 per cent higher than was the ac.

The maximum expected experimental error for ac measurements is 0.19 per cent, compared to 0.82 per cent for dc, based on these results.

A second set of measurements made by C. L. Paulnack and S. J. Silverman is shown in Fig. 10, for a floating-zone silicon rectilinear crystal, approaching intrinsic resistivity of 11,000 ohm-cm. This 11,000

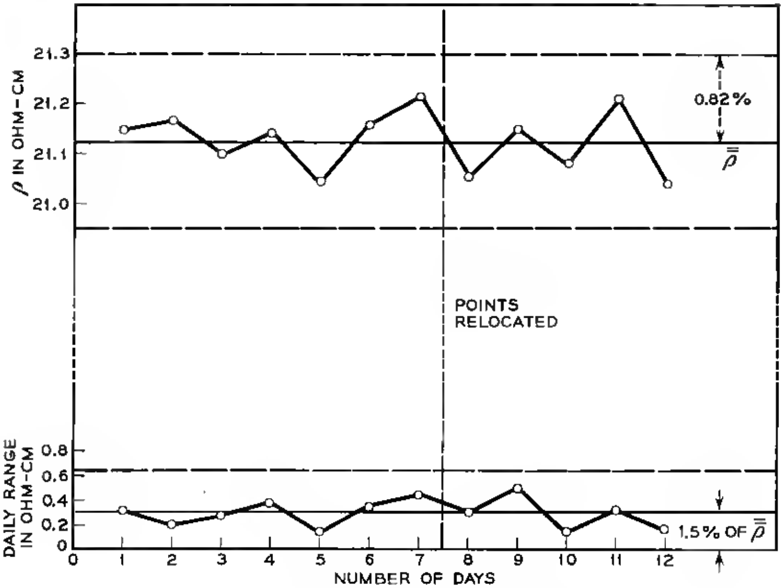


Fig. 8 — Two-point dc resistivity measurements of silicon.

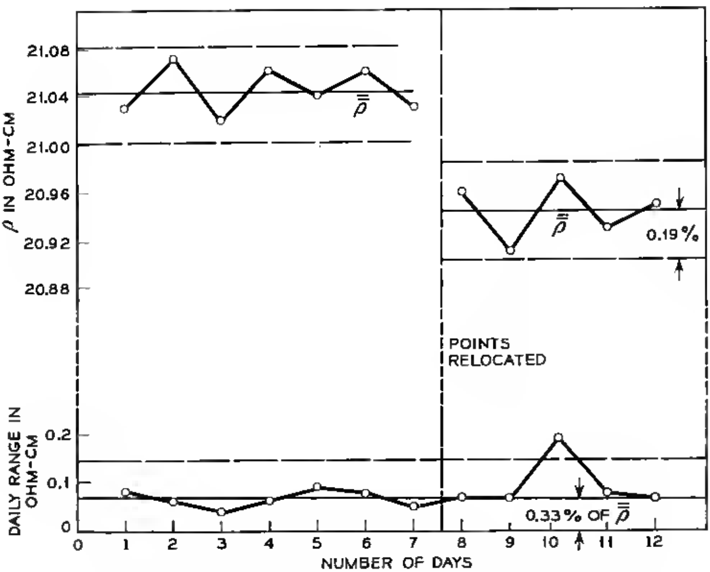


Fig. 9 — Two-point ac resistivity measurements of silicon.

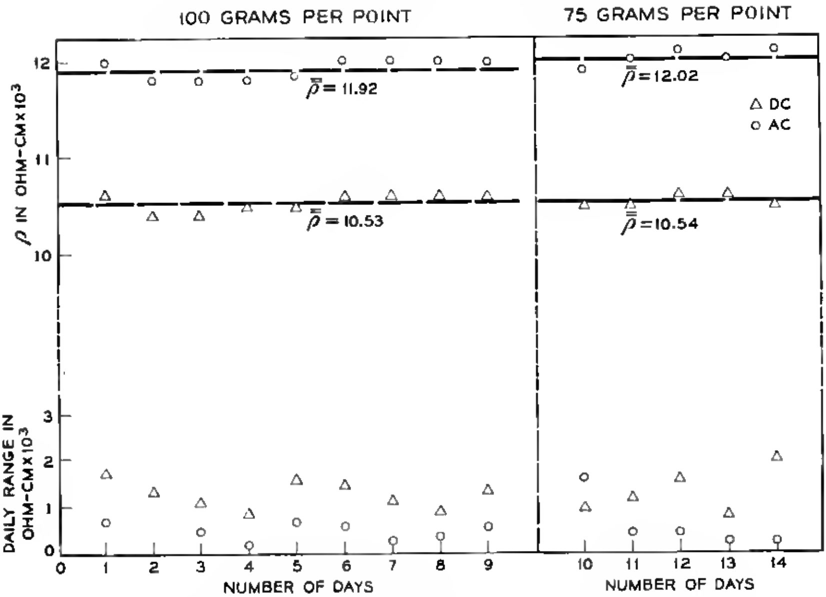


Fig. 10 — Resistivity of floating-zone-refined silicon.

ohm-cm material is approaching the upper design limit for the ac set, but the range is still about one-third that for the dc set. The dc resistivity indication was 14 per cent lower than was the ac.

Independence of sheet resistivity to point force is not true for very thin diffused junctions. With elastic deformation, the number of intrinsic carriers present is increased, the effect diminishing with distance from the needle point. These intrinsic carriers subtract from the effect of the impurities present, which formed the junction. If the junction is close enough to the surface, the effect of increasing the force is to observe an apparent abrupt decrease in sheet resistivity, as deformation of the junction brings it to the surface and conduction through the body then also obtains. This effect is reversible.

15.2 *Four-Point Probe*

Fig. 11 shows measurements on slices from an n-type silicon ingot using a very stable four-point probe. The probe design is based on a development due to N. J. Chaplin. Each slice was measured four times, with the points being raised and lowered between each measurement. These measurements include two causes for error, electrical and mechani-

cal. The relative position of the voltage needle points to the current needle points cannot absolutely be fixed, because of the requirement of independent and equal force application to all four points. The pairs are suspended independently, which permits relative pair position changes. However, any error is minimized, because half-way along a line between the two current points the voltage gradient has a broad minimum. Hence, the percentage error in voltage difference is less than the percentage error in mechanical displacement from the ideal location.

From Fig. 11, the short-time average range for four measurements is 0.65 per cent of the resistivity, including both electrical and point-spacing variations. From this, the maximum experimental error is 0.47 per cent. It can be verified independently that most of the variations are due to point spacing, by making repeated impressions of the points on a polished lead sheet and then measuring the actual spacings with a machinist's microscope. Over a longer time, the point-spacing variations can be expected to increase further, reducing the over-all precision. Point wear is one effect, but a more important cause is permanent relative changes in spacing due to handling of the probe assembly. In view of the results of the preceding section, it is evident that the electrical accuracy is superior to the point-spacing accuracy, even with the very stable four-point probe used in these tests. Hence, an evaluation of only electrical precision with a four-point probe cannot be achieved, only the over-all precision, as shown. Even so, the expected experimental error will be less than 0.5 per cent.

In general, ac measurements using device material of less than 500

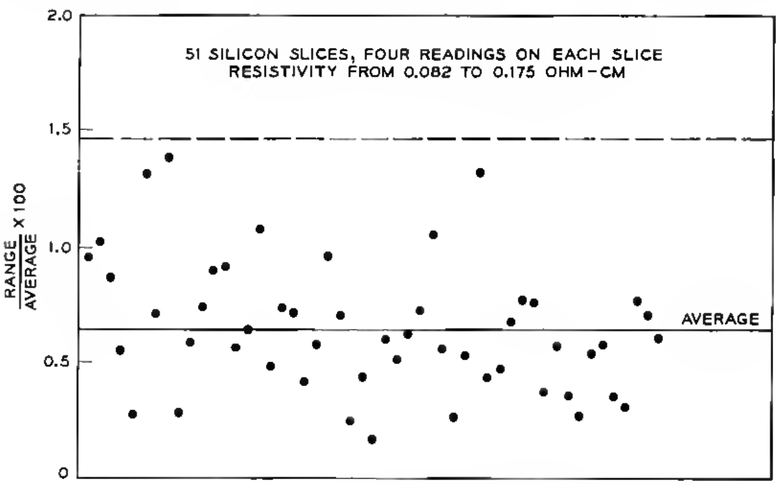


Fig. 11 — Four-point probe measurements.

ohm-cm resistivity are 5 to 15 per cent lower in resistivity than dc measurements. This consistent difference has not been accounted for.

XVI. EQUIPMENT

A photograph of the test set is shown in Fig. 12. The lower panel contains the resistance decades, the decade potentiometer, the bridge

Fig. 12 — Front view of semiconductor resistivity bridge.

and its zero balancing controls, the first stage of the preamplifier, and the four precision amplifiers. The center panel contains the second stage of the preamplifier, a 390-cycle bandpass filter, an ac millivoltmeter, and the regulated dc heater supply for the input tubes of the precision amplifiers. The upper panel includes the power supplies, the ac oscillator and test current controls, and a dc voltmeter for monitoring several circuits. The oscillator may be operated at either 390 cycles or 85 cycles. For all ordinary device work, a frequency of 390 cycles can be used, the bridge is self-contained, and only a probe is needed as auxiliary apparatus. When a frequency of 85 cycles is necessary, an external 4-cycle bandpass wave analyzer must be provided.

In Fig. 13 the probe amplifier and control circuit is shown affixed to the side of a four-point probe. The input vacuum tubes of the $+V_1$ and $+V_2$ amplifiers are underneath. Terminals are on top to connect direct wires to the needle points. One switch selects for measurement, bridge balance or calibrate, using a simulating resistor network described earlier. The other selects for use of a two- or four-point probe. Four-foot cables connect this circuit to the back of the resistivity bridge chassis.

XVII. CONCLUSION

This report has described the principles used in the development of a new general-purpose semiconductor resistivity measuring set using a four- or two-point probe and having an over-all electrical accuracy of better than 0.5 per cent. It covers the range from 0.001 to 10,000 ohm-cm material. It is an ac bridge, and every component is grounded except the specimen and the four-point probe.

The precision and accuracy, compared to a DC measurement, result from the fact that, while surface and point-contact potential effects cannot be separated from a dc measurement, they cannot affect the fundamental frequency ac voltage-current ratio. This principle is the basis for the measuring set design. In utilizing this principle, and at the same time grounding all components to permit ac power operation, a bridge circuit has been developed. The successful realization of the precision and accuracy required the development of an "open grid" differential amplifier that is both linear and has a common mode suppression of one part in a million.

APPENDIX

A.1 *Sheet Resistivity Conversion Factors*

When measurements are made with a four-point probe on thin homogeneous slices or surfaces having a one-sided diffused junction, the

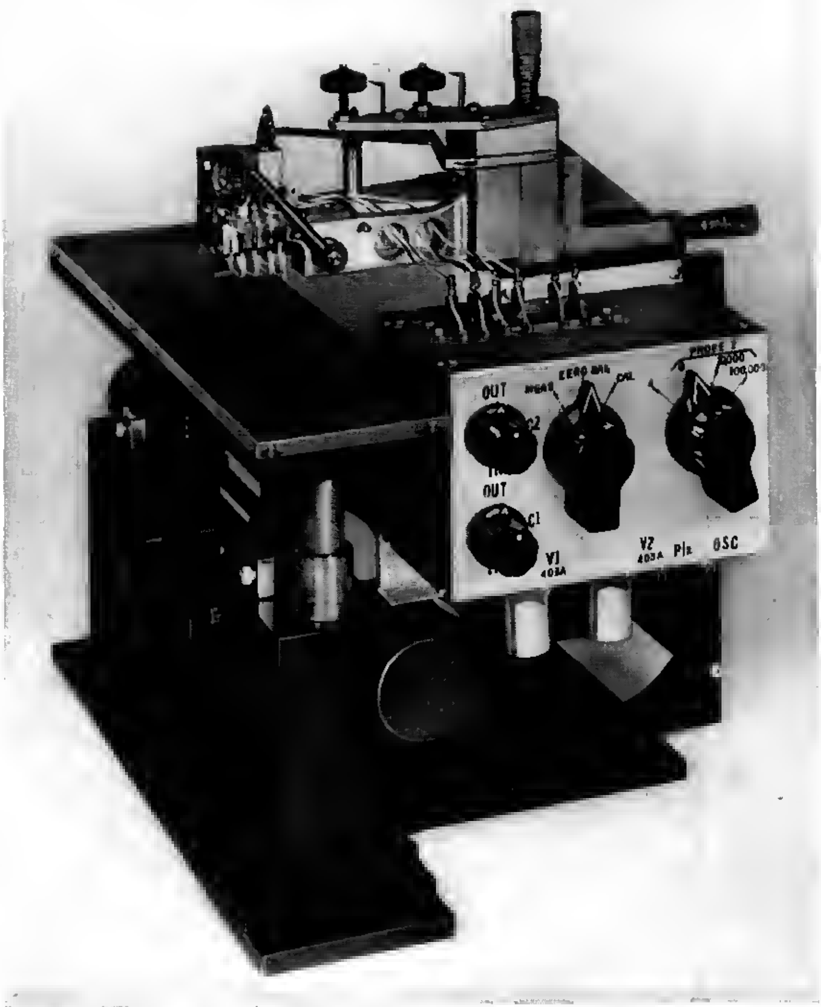


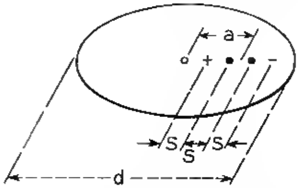
Fig. 13 — Four-point probe with amplifier attached.

resistivity reading V/I is altered, compared to an infinite sheet, by the finite size of the sample. Even for an infinite sheet, a conversion factor $\pi/(\ln 2)$ applies for equally spaced points on a line. Multiplying a measured V/I reading by the appropriate conversion factor places all measurements on a common, infinite-sheet basis. That is:

$$\rho_s = \frac{V}{I} \times \text{conversion factor.}$$

TABLE I — CONVERSION FACTORS FOR SHEET RESISTIVITY MEASUREMENTS OF CIRCULAR SAMPLE USING FOUR-POINT PROBE

Sheet with Insulated Edges



$0 \le a \le \left(\frac{d}{2} - \frac{3S}{2}\right)$

POINTS ON DIAMETER

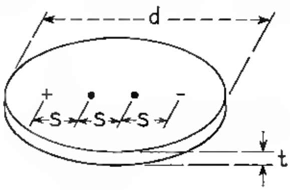
$\rho_S = \frac{V}{I}\text{C.F.};\ \rho \doteq \frac{V}{I}\text{C.F.W};\ W/S < 0.5$

Conversion Factor =

$$\frac{\pi}{\ln 2 + \frac{1}{2}\ln \dfrac{\left[1 - \left(\frac{2a}{d} + \frac{S}{d}\right)\left(\frac{2a}{d} - \frac{3S}{d}\right)\right]\left[1 - \left(\frac{2a}{d} - \frac{S}{d}\right)\left(\frac{2a}{d} + \frac{3S}{d}\right)\right]}{\left[1 - \left(\frac{2a}{d} - \frac{S}{d}\right)\left(\frac{2a}{d} - \frac{3S}{d}\right)\right]\left[1 - \left(\frac{2a}{d} + \frac{S}{d}\right)\left(\frac{2a}{d} + \frac{3S}{d}\right)\right]}}$$

d/S	$a/d = 0$	$a/d = 0.1$	$a/d = 0.2$	$a/d = 0.3$	$a/d = 0.4$	$a/d = 0.45$
3	2.2662					
4	2.9289					
5	3.3625	3.2719	2.9176			
7.5	3.9273	3.8780	3.6903	3.1123		
10	4.1716	4.1415	4.0263	3.6754		
15	4.3646	4.3504	4.2957	4.1284	3.2635	
20	4.4364	4.4282	4.3967	4.3000	3.8038	
40	4.5076	4.5059	4.4977	4.4730	4.3451	3.8568
∞	4.5324	4.5324	4.5324	4.5324	4.5324	4.5324

Conducting Sheet on All Surfaces



$2W < t \ll d$

W = JUNCTION DEPTH

$\frac{d+t}{S}$	Conversion Factor $= \frac{\pi}{\ln 2}$ (also independent of location of sample)
3	4.5324
4	4.5324
5	4.5324
7.5	4.5324
10	4.5324
15	4.5324
20	4.5324
40	4.5324
∞	4.5324

When slices have a continuous diffused skin all over, however, conversion terms more nearly approaching $\pi/(\ln 2)$ (the infinite sheet case) apply. Values for some important practical cases have been calculated, and tabulated along with Smits' earlier figures,[2] for ready reference.

A.2 *Circular Sample*

For a one-sided diffusion, only one image is necessary for each current point, to fulfill the boundary condition. Furthermore, the points need not be symmetrically placed with respect to the center. An extension of Smits' table for the circle, for the case of the four equally spaced points lying along a diameter, has been calculated and is shown in Table I.

For a two-sided diffusion the presence of conduction across the back surface acts for a circle exactly as if the front were part of a continuous infinite sheet. Hence, the conversion factor for this important case is always the same no matter where the points are placed or arranged.

The proof for the two circular conducting sheets, connected at the circumference, can be obtained by using conformal transformations. First cut the circumference, unfold, and place in the coordinate system as shown in Fig. 14. Then transform the upper circle into the upper half

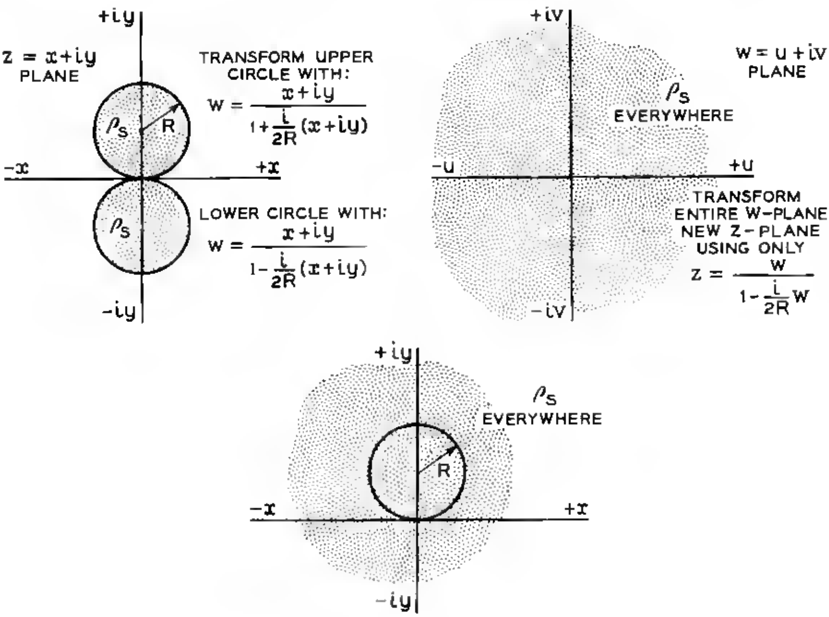
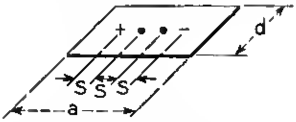


Fig. 14 — Conformal transformations for double-surfaced circle.

Table II — Conversion Factors for Sheet Resistivity Measurements of Rectangular Sample Using Four-Point Probe

Sheet with Insulated Edges

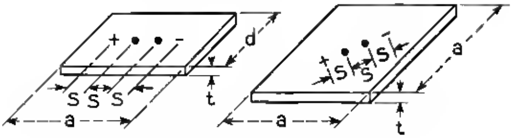


$$\rho_S = \frac{V}{I} C.F.;\ \rho \doteq \frac{V}{I} C.F. W;\ W/S < 0.5$$

W = JUNCTION DEPTH

d/S	$a/d = 1$	$a/d = 2$	$a/d = 3$	$a/d \geqq 4$
1.0			0.9988	0.9994
1.25			1.2467	1.2248
1.5		1.4788	1.4893	1.4893
1.75		1.7196	1.7238	1.7238
2.0		1.9454	1.9475	1.9475
2.5		2.3532	2.3541	2.3541
3.0	2.4575	2.7000	2.7005	2.7005
4.0	3.1137	3.2246	3.2248	3.2248
5.0	3.5098	3.5749	3.5750	3.5750
7.5	4.0095	4.0361	4.0362	4.0362
10.0	4.2209	4.2357	4.2357	4.2357
15.0	4.3882	4.3947	4.3947	4.3947
20.0	4.4516	4.4553	4.4553	4.4553
40.0	4.5120	4.5129	4.5129	4.5129
∞	4.5324	4.5324	4.5324	4.5324

Conducting Sheet on All Surfaces



$$\rho_S = \frac{V}{I} C.F.;\ W < t/2$$

$\frac{d+t}{S}$	$\frac{a+t}{d+t} = 1$	$\frac{a+t}{d+t} = 2$	$\frac{a+t}{d+t} = 3$	$\frac{a+t}{d+t} \geqq 4$	$\frac{a+t}{S}$	Square Measure on Diagonal
1.0			1.9976	1.9497		
1.25			2.3741	2.3550		
1.5		2.9575	2.7113	2.7010		
1.75		3.1596	2.9953	2.9887		
2.0		3.3381	3.2295	3.2248	2.0	3.4700
2.5		3.6408	3.5778	3.5751	2.5	3.8696
3.0	4.9124	3.8543	3.8127	3.8109	3.0	4.1943
4.0	4.6477	4.1118	4.0899	4.0888	4.0	4.4212
5.0	4.5790	4.2504	4.2362	4.2356	5.0	4.4865
7.5	4.5415	4.4008	4.3946	4.3943	7.5	4.5233
10.0	4.5353	4.4571	4.4536	4.4535	10.0	4.5295
15.0	4.5329	4.4985	4.4969	4.4969	15.0	4.5318
20.0	4.5326	4.5132	4.5124	4.5124	20.0	4.5322
40.0	4.5325	4.5275	4.5273	4.5273	40.0	4.5323
∞	4.5324	4.5324	4.5324	4.5324	∞	4.5324

of a w plane and the lower circle into the lower half, reconnecting the two, now semi-infinite, surfaces along the x axis. The transformations shown are chosen to keep the origin in the center, and to place $i2R$ of the upper circle and $-i2R$ of the lower circle at infinity on the real axis. The associated overlapping insulating sheets formed by the areas outside the circles are discarded. Finally, transform the entire w-plane back into a new z-plane using everywhere the inverse transformation which restores the upper half of the w-plane to the original upper circle. The lower half of the w-plane fills the entire new z-plane outside the circle, and is connected to it at the circumference. Thus, in the new z-plane, any measurement inside the circle, which is merely a part of an infinite sheet, is identical with the same measurement on the original surface.

A.3 *Rectangular Sample*

For a one-sided diffusion, as shown by Smits and included in Table II, a doubly infinite array of image point pairs, one pair in each of the identical rectangles covering an infinite sheet, represents the system. For a two-sided diffusion covering all surfaces of the slice, image point pairs of the equivalent infinite sheet appear in only half the rectangles, checkerboard fashion, to represent the system. The rectangles void of image points represent the conducting under side.

The image point array for a rectangle with a two-sided diffusion is shown in Fig. 15. All points contribute to the voltage between points 1 and 2.

Each horizontal infinite line of equally spaced current sources, a distance $2d$ apart, as shown by Ollendorff[4] and diagrammed by Smits, causes a potential of

$$\varphi - \varphi_0 = -\frac{I\rho_s}{2\pi}\ln\left(2\sinh\frac{\pi y}{2d}\right)$$

when a perpendicular to the line passes both through the voltage point and a current source, and

$$\varphi - \varphi_0 = -\frac{I\rho_s}{2\pi}\ln\left(2\cosh\frac{\pi s}{2d}\right)$$

when a perpendicular to the line passes through the voltage point but is half-way between two of the current sources. For both cases, the perpendicular distance y is measured from the line of current sources to the voltage point being evaluated. For a current sink, the sign is reversed.

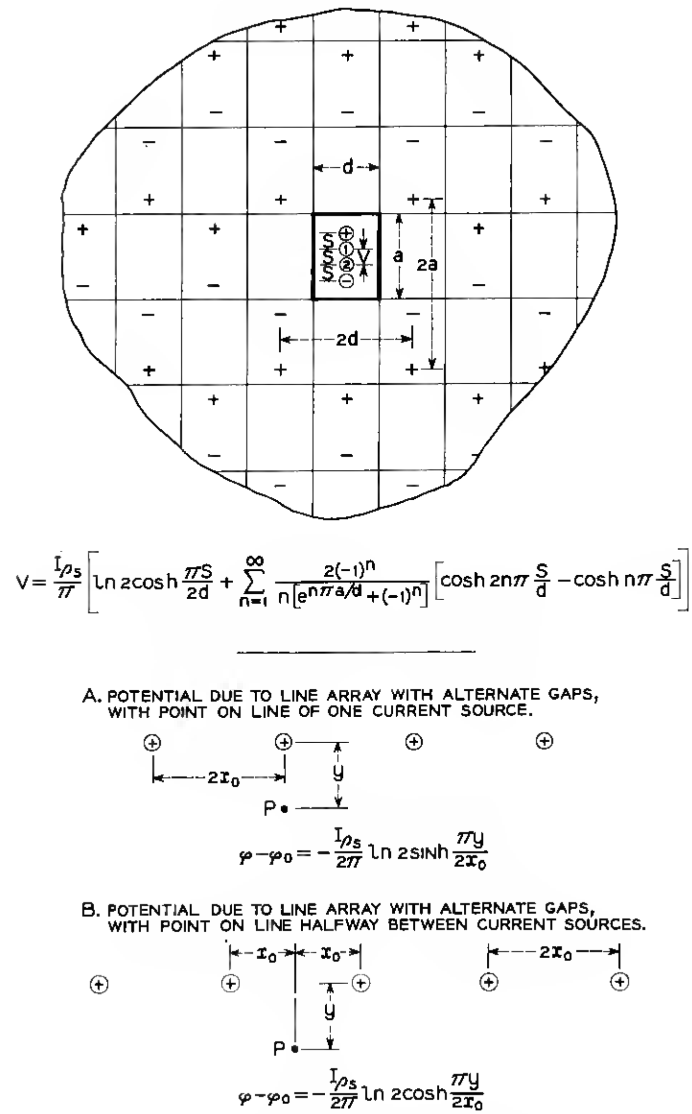


Fig. 15 — Image point array for rectangle with two-sided diffusion.

These expressions reduce the present problem to a summation of potentials, each due to a line of current sources, in only one direction.

Taking separately the two lines of current sources which include the two real current sources, their contribution to the desired total is

$$\Delta V = \frac{I\rho_s}{\pi} \ln\left(2 \cosh \frac{\pi s}{2d}\right).$$

Now, counting up all the other lines, eight infinite series are formed:

$$\Delta V = -\frac{I\rho_s}{\pi} \sum_{n=1}^{\infty} \left[\begin{array}{l} +\ln\left(2\cosh\frac{\pi}{2d}\right)[(2n-1)a-2s] \\ +\ln\left(2\cosh\frac{\pi}{2d}\right)[(2n-1)a+2s] \\ +\ln\left(2\sinh\frac{\pi}{2d}\right)(2na+s) \\ +\ln\left(2\sinh\frac{\pi}{2d}\right)(2na-s) \\ -\ln\left(2\cosh\frac{\pi}{2d}\right)[(2n-1)a-s] \\ -\ln\left(2\cosh\frac{\pi}{2d}\right)[(2n-1)a+s] \\ -\ln\left(2\sinh\frac{\pi}{2d}\right)(2na+2s) \\ -\ln\left(2\sinh\frac{\pi}{2d}\right)(2na-2s) \end{array} \right].$$

The hyperbolic terms are changed to the exponential forms and the logarithms of the products separated to the sum of two logarithms. The first term above thus becomes:

$$\frac{\pi}{2d}[(2n-1)a-2s] + \ln\left[1 + e^{-\pi[(2n-1)(a/d)-2(s/d)]}\right].$$

The sum of the first terms of the eight expressions thus formed cancels. The eight logarithms are placed in series form, the first one being

$$e^{-\pi[(2n-1)(a/d)-2(s/d)]} - \tfrac{1}{2}\left(e^{-\pi[(2n-1)(a/d)-2(s/d)]}\right)^2 + \tfrac{1}{3}(\cdots.$$

The first terms of the eight series now are summed, then the second terms, etc. The condensed form for the first term is:

$$2\left(e^{-\pi(2n-1)(a/d)} + e^{-\pi 2n(a/d)}\right)\left(\cosh 2\pi\frac{s}{d} - \cosh\pi\frac{s}{d}\right).$$

The numbers $n = 1, 2, 3, 4, \cdots$ are next substituted in each such expression, which is factored, and the geometric series is identified and summed, yielding one final series:

$$\Delta V = \frac{I\rho_s}{\pi} \sum_{n=1}^{\infty} \frac{2(-1)^n}{n[e^{n\pi(a/d)} + (-1)^n]} \left(\cosh 2n\pi \frac{s}{d} - \cosh n\pi \frac{s}{d}\right).$$

Adding the ΔV term for the two separately evaluated lines and rearranging:

$$\rho_s = \frac{V}{I} \times \text{conversion factor (C.F.)},$$

where

$$\text{C.F.} = \frac{\pi}{\ln\left(2\cosh\frac{\pi s}{2d}\right) + \sum_{n=1}^{\infty} \frac{2(-1)^n}{n[e^{n\pi(a/d)} + (-1)^n]}\left(\cosh 2n\pi\frac{s}{d} - \cosh n\pi\frac{s}{d}\right)}.$$

A.4 *Double-Sided Square Sample With Points on a Diagonal*

A square sample with points on a diagonal can be cut to form an equivalent single sheet, as shown in Fig. 16. First observe that the horizontal diagonal, both front and back, is an equipotential line. Any point on it is equidistant from the current source and sink. Second, no current crosses the vertical diagonal, both front and back, because of symmetry. Cut these two diagonals on the back, unfold, and make the edges parallel to the line of points conducting to maintain the equipotential condition, as in Fig. 16. The other two edges are nonconducting.

This problem also can he solved by the method of images. The doubly infinite array which represents this problem is shown on Fig. 17. Each horizontal infinite line of equally spaced but alternate current sources

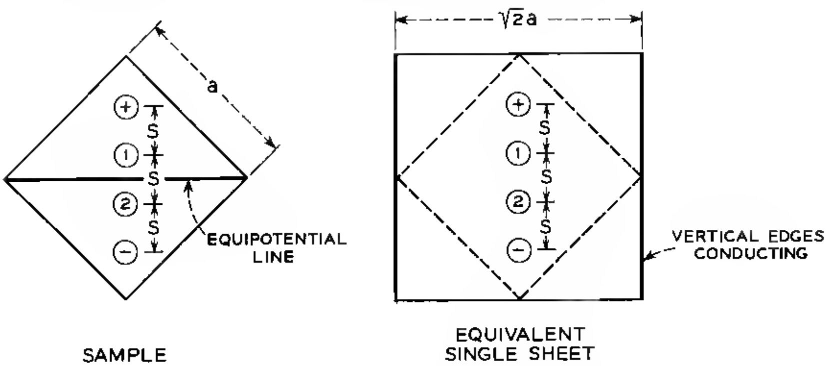


Fig. 16 — Square with front and back conduction and points on a diagonal.

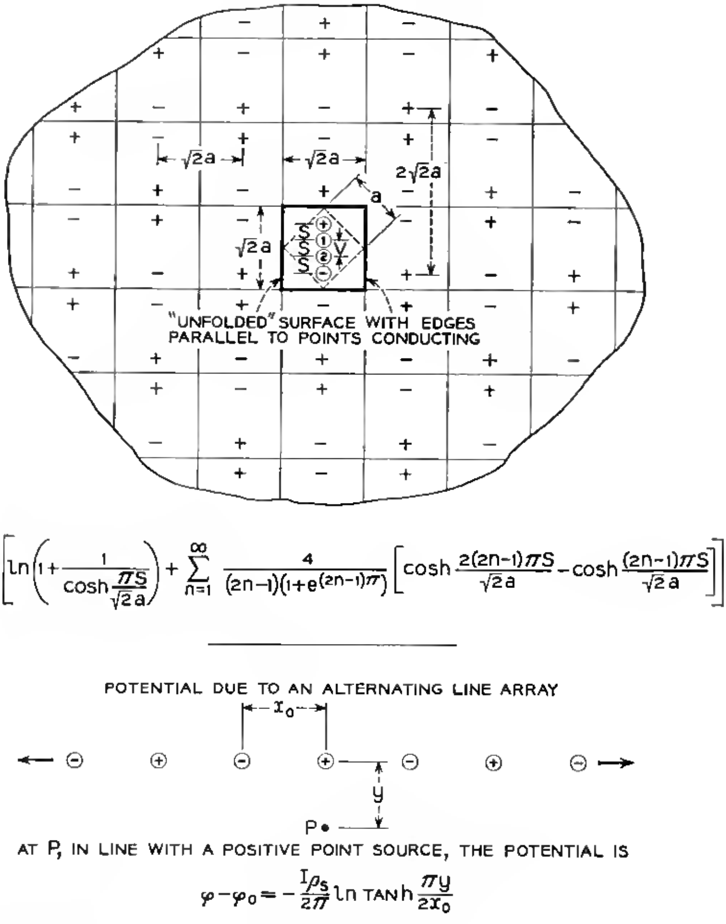


Fig. 17 — Image point array for square with two-sided diffusion and points on a diagonal.

and sinks, a distance $\sqrt{2}a$ apart, using a combination of the two line potential equations of the previous example, causes a potential of

$$\varphi - \varphi_0 = -\frac{I\rho_s}{2\pi}\ln\tanh\frac{\pi y}{2\sqrt{2}a}$$

at a point a perpendicular distance y from a source. When in line with a sink, the sign is reversed.

As before, this expression reduces the problem to a summation of potentials, each due to a line of current sources and sinks, in only one direction.

Taking separately the two lines that include the real current source and sink, their contribution to the desired total is

$$\Delta V = \frac{I\rho_s}{\pi} \ln \left[1 + \frac{1}{\cosh \frac{\pi s}{\sqrt{2}a}} \right].$$

Counting up all the other lines, eight infinite series are again found:

$$\Delta V = \frac{I\rho_s}{\pi} \sum_{n=1}^{\infty} \left[\begin{array}{l} +\ln \dfrac{1}{\tanh \pi \left(n + \dfrac{s}{2\sqrt{2}a}\right)} \\ +\ln \dfrac{1}{\tanh \pi \left(n - \dfrac{s}{2\sqrt{2}a}\right)} \\ -\ln \dfrac{1}{\tanh \pi \left(n + \dfrac{s}{\sqrt{2}a}\right)} \\ -\ln \dfrac{1}{\tanh \pi \left(n - \dfrac{s}{\sqrt{2}a}\right)} \\ +\ln \dfrac{1}{\tanh \pi \left[\left(n - \dfrac{1}{2}\right) + \dfrac{s}{\sqrt{2}a}\right]} \\ +\ln \dfrac{1}{\tanh \pi \left[\left(n - \dfrac{1}{2}\right) - \dfrac{s}{\sqrt{2}a}\right]} \\ -\ln \dfrac{1}{\tanh \pi \left[\left(n - \dfrac{1}{2}\right) + \dfrac{s}{2\sqrt{2}a}\right]} \\ -\ln \dfrac{1}{\tanh \pi \left[\left(n - \dfrac{1}{2}\right) - \dfrac{s}{2\sqrt{2}a}\right]} \end{array} \right].$$

Convert these to exponential form using

$$\frac{1}{\tanh x} = \frac{1 + e^{-2x}}{1 - e^{-2x}}$$

and then expand the logarithm into a series. The first term becomes:

$$2[e^{-2\pi[n+(s/2\sqrt{2}a)]} + \tfrac{1}{3}(e^{-2\pi[n+(s/2\sqrt{2}a)]})^3 + \tfrac{1}{5}(\cdots].$$

Add the first terms of the first four, the first terms of the second four; then the second terms of the first four, the second terms of the second four; etc. Factor out the common terms in each group of four, rearrange and add the pairs, ending with a single series. The first term is

$$4(e^{-\pi(2n-1)} - e^{-2\pi n})\left(\cosh\frac{2\pi s}{\sqrt{2}a} - \cosh\frac{\pi s}{\sqrt{2}a}\right).$$

Substitute $n = 1, 2, 3$, etc., into the first bracket, factor out the common terms, identify the geometric series, and sum, yielding the final series:

$$\Delta V = \frac{I\rho_s}{\pi}\sum_{n=1}^{\infty}\frac{4}{(2n-1)(1+e^{(2n-1)\pi})} \cdot\left[\cosh\frac{2(2n-1)\pi s}{\sqrt{2}a} - \cosh\frac{(2n-1)\pi s}{\sqrt{2}a}\right].$$

Adding the ΔV term for the two separately evaluated lines and rearranging:

$$\rho_s = \frac{V}{I} \times \text{C. F.},$$

where

$$\text{C.F.} = \frac{\pi}{\ln\left(1 + \dfrac{1}{\cosh\dfrac{\pi s}{\sqrt{2}a}}\right) + \displaystyle\sum_{n=1}^{\infty}\frac{4}{(2n-1)(1+e^{(2n-1)\pi})} \cdot\left[\cosh\frac{2(2n-1)\pi s}{\sqrt{2}a} - \cosh\frac{(2n-1)\pi s}{\sqrt{2}a}\right]}.$$

REFERENCES

1. Valdes, L. B., Resistivity Measurements on Germanium for Transistors, Proc. I.R.E., **42,** 1954, p. 420.
2. Smits, F. M., Measurement of Sheet Resistivities with the Four-Point Probe, B.S.T.J., **37,** 1958, p. 699.
3. Holm, R., *Electric Contacts* (Eng. trans.), Hugo Gebers Förlag, Stockholm, 1946.
4. Ollandorff, F., *Potentialfelder der Elektrotechnik*, Springer, Berlin, 1932.